

Before the
Federal Communications Commission
Washington, DC 20554

In the Matter of)	
)	
Unlicensed Operation in the TV Broadcast Bands)	ET Docket No. 04-186
)	
Additional Spectrum for Unlicensed Devices Below 900 MHz and in the 3 GHz Band)	ET Docket No. 02-380
)	

NOTICE OF PROPOSED RULE MAKING AND ORDER FCC 04-113

Comments by Adaptrum, Inc.

Adaptrum, Inc. is submitting these comments in response to the Notice of Proposed Rule Making and Order issued in the above captioned proceeding.

Adaptrum, Inc. is a newly formed company developing spectrum sharing technologies. In our comments, we intend to supply some measurement results regarding spectrum opportunity in the TV bands and address some of the technical issues raised in the NPRM proceeding. It is our sincere hope that the comments provided herein can help advance the understanding on the subject matter and assist FCC in any way in this historical ruling.

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1 Introduction

We strongly support the subject NPRM regarding unlicensed operation in the TV bands. This NPRM together with the earlier NPRM on cognitive radio technologies [1] constitutes a historical step on FCC's part towards a new paradigm in spectrum sharing. The importance of this ruling is evident from the following measures:

- Economic impact. Allowing the spectrum sharing in the TV bands effectively releases hundreds of megahertz worth of bandwidth from the bands which are currently underutilized due to outdated technology. Such amount of bandwidth in the bands with superior propagation characteristics will create multi-billion dollar industries providing related products and services to benefit U.S. businesses and consumers.
- Policy impact. Allowing dynamic spectrum sharing represents a paradigm shift in FCC's spectrum policy. Spectrum sharing in the TV bands is the first step in this direction where the experience learned could help shape the future spectrum policy.
- Technology impact. Challenges in dynamic spectrum sharing will spur technology innovations in sensing, wideband transceivers, cross-layer coordination, and network collaboration. The transition from static spectrum sharing to dynamic spectrum sharing could well be a wireless technology revolution reminiscent of that in the wired world from circuit-switched telephony network to Internet.

We will focus our comments on technical discussions outlined as following:

1. We will report a measurement survey regarding the TV signal coverage in the Bay Area through which we assess the TV band spectrum opportunities.
2. We will provide a theoretical framework to analyze the hidden terminal problem and quantify the problem in terms of hidden terminal probability. We then discuss how collective sensing by a network of cognitive users could reduce the hidden terminal probability.
3. We conclude our comments by summarizing our observations and making certain recommendations regarding the NPRM.

2 Bay Area TV signal coverage survey and spectrum opportunity assessment

2.1 Signal coverage survey

Figure 1 shows the topographical view of the Bay Area survey. Each empty red triangle represents a survey location. Filled blue triangles are possible TV station locations obtained from FCC TV database query. At each survey location, a frequency scan over all the TV bands is performed using a spectrum analyzer while the precise survey position is recorded by a GPS. We have been about one survey location in every two miles and a total of 240 survey locations in populated areas around the Bay.

Figure 2 shows the spectrum measurement setup. The antenna is an omnidirectional wideband discone antenna with 2dBi gain. The RF filter, which has an insertion loss less than 1dB, is used to reject the FM radio signal and cellular signal which otherwise may saturate the measurement system at certain locations. Two cascaded broadband amplifiers with a combined gain around 30dB and noise figure around 4.5dB are used to preamplify the signal coming into the spectrum analyzer – so as to overcome the high noise figure of the spectrum analyzer which is about 30-40dB in our case. The overall system noise figure considering all components is less than 10dB. We use a resolution bandwidth of 10kHz for the spectrum scan at each survey location. The scanned power values within a 6MHz TV channel are averaged and scaled by $10 \log_{10}(6\text{MHz}/10\text{kHz}) = 27.8\text{dB}$ to give the total signal power in the channel.

Although the measurements were taken on different days, we have observed remarkable consistency in terms of the measured signal coverage contours over the region. Figure 3 shows the signal coverage map for Channel 14. The signal appears to be coming from station KDTV in San Jose area. Figure 4 shows the signal coverage map for Channel 19 whose signal appears to be coming from station KBWB in San Francisco. In both cases, we can see significant terrain blocking – while the signal can easily reach most of the (low-altitude) areas around the Bay, it is unable to penetrate the coastal hills to reach inland areas like Pleasanton, San Ramon, Walnut Creek, Concord, etc.

2.2 Spectrum opportunity assessment

The spectrum opportunity at each survey location is defined by the number of *vacant* TV channels. Criterion must be established to determine the channel vacancy.

The data rate of terrestrial DTV system is 19.39Mbps [2]. Since the effective bandwidth of the DTV system is 5.38MHz [2], the threshold signal-to-noise ratio SNR_T for successful DTV signal decoding based on Shannon capacity is

$$C = W \log_2(1 + \text{SNR}_T) \Rightarrow \text{SNR}_T = 2^{C/W} - 1 = 2^{19.39/5.38} - 1 \approx 10.5\text{dB}$$

Practical systems are usually at least 4-5dB [3] away from the Shannon limit. Thus a 15dB SNR_T would be required for the DTV system. Note that to account for SNR degradations

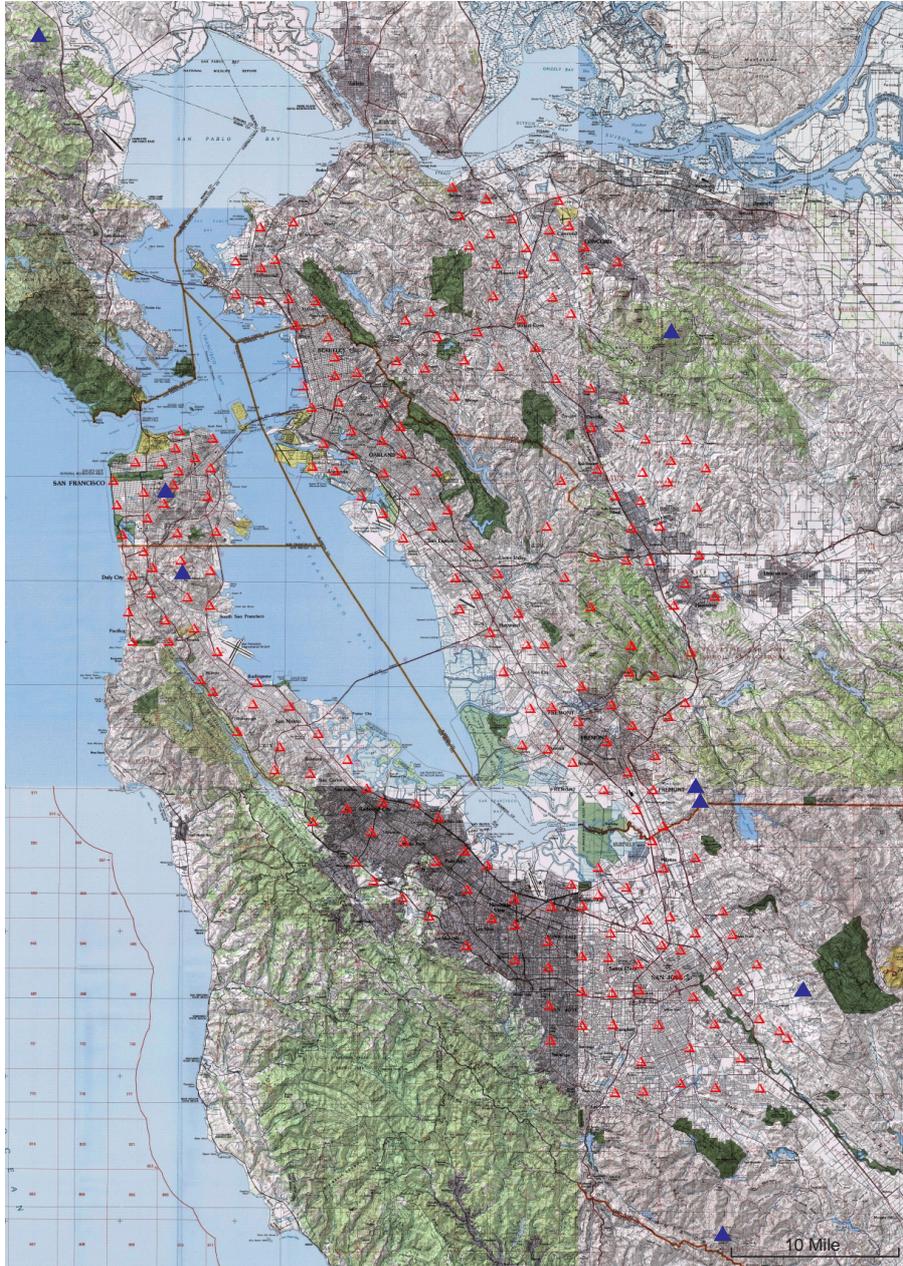


Figure 1: Topographical view of the Bay Area survey. Empty red triangles are survey locations. Filled blue triangles are possible TV station locations.

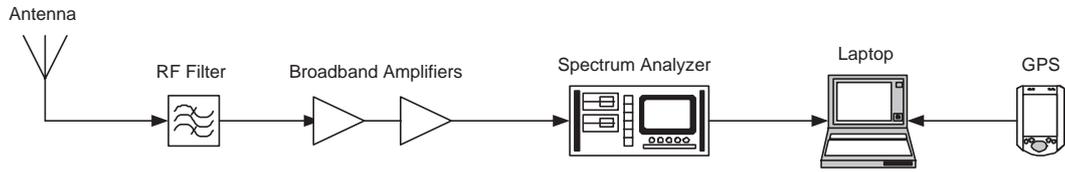


Figure 2: TV signal measurement setup.

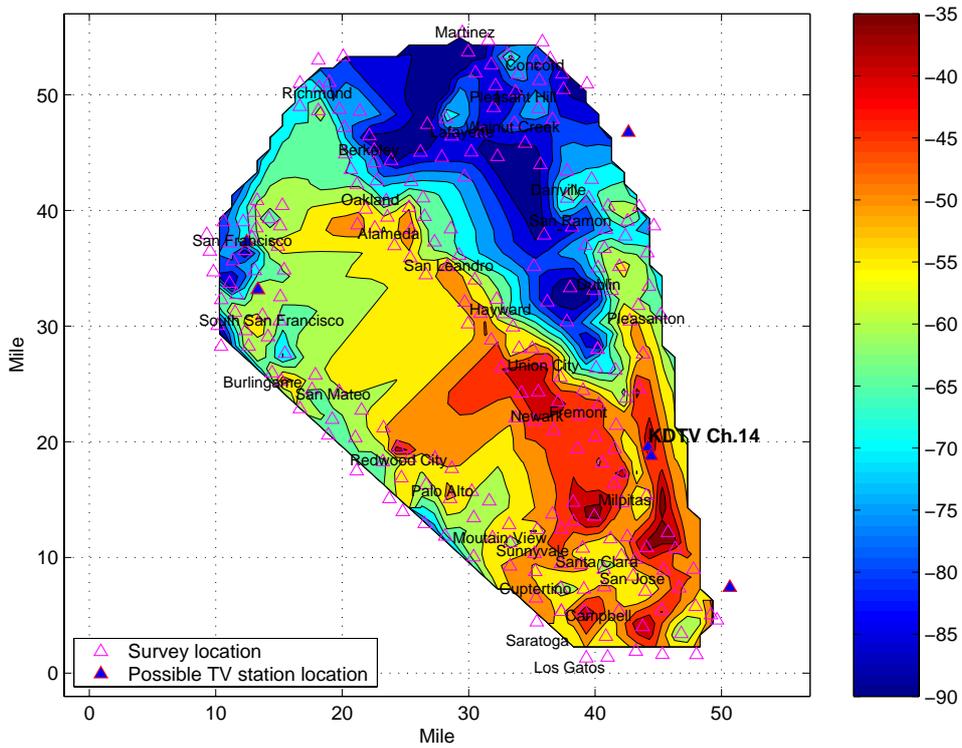


Figure 3: Channel 14 (470-476MHz) Bay Area signal coverage. Signal strength on the colorbar is in dBm.

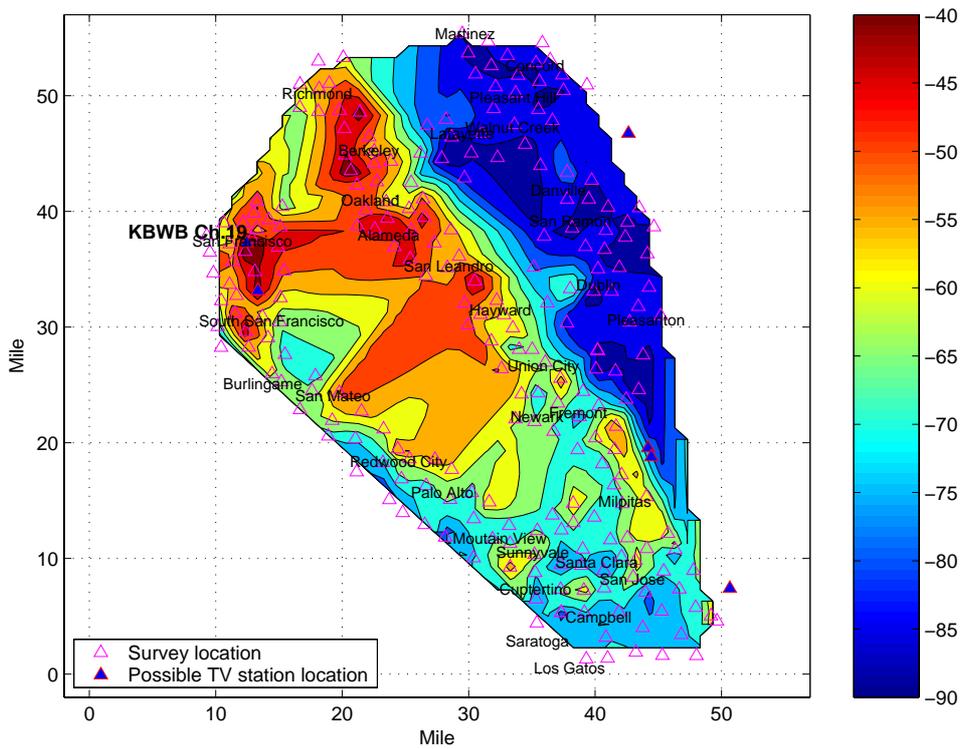


Figure 4: Channel 19 (500-506MHz) Bay Area signal coverage. Signal strength on the colorbar is in dBm.

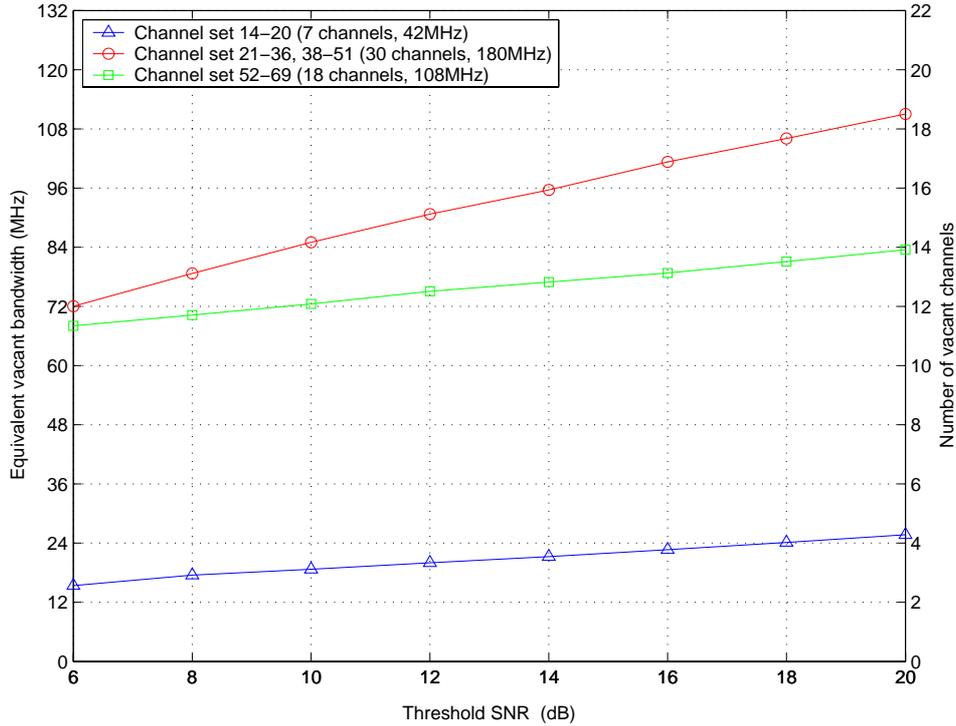


Figure 5: Average number of vacant channels for different bands over all survey locations.

due to multipath, analog impairments, and digital implementation loss could result in considerably higher SNR_T . We assume the NTSC system will require an even higher threshold SNR than the DTV system.

We defined the TV channel *vacancy threshold* as

$$V_T = N_0 + \text{NF} + \text{SNR}_T$$

where N_0 is the thermal noise floor; NF is the receiver noise figure; all in dB units. The thermal noise floor for a 6MHz TV channel is about -106dBm. A typical TV receiver has a noise figure of 10dB [4]. If a 15dB threshold SNR is used, the vacancy threshold V_T is $-106 + 10 + 15 = -81$ dBm. When the received signal power on the channel is less than the vacancy threshold, the TV receiver will not be able to successfully decode the signal and the channel is considered as vacant.

Since our measurement system has the same noise figure as a typical TV receiver, the vacancy threshold is determined by selecting the threshold SNR. Figure 5 plots the average number of vacant channels for different bands over all survey locations as a function of the threshold SNR.

It is more instructive to see the geographic distribution of channel usage. A channel is considered to be used if it has signal power above the vacancy threshold (and it is vacant

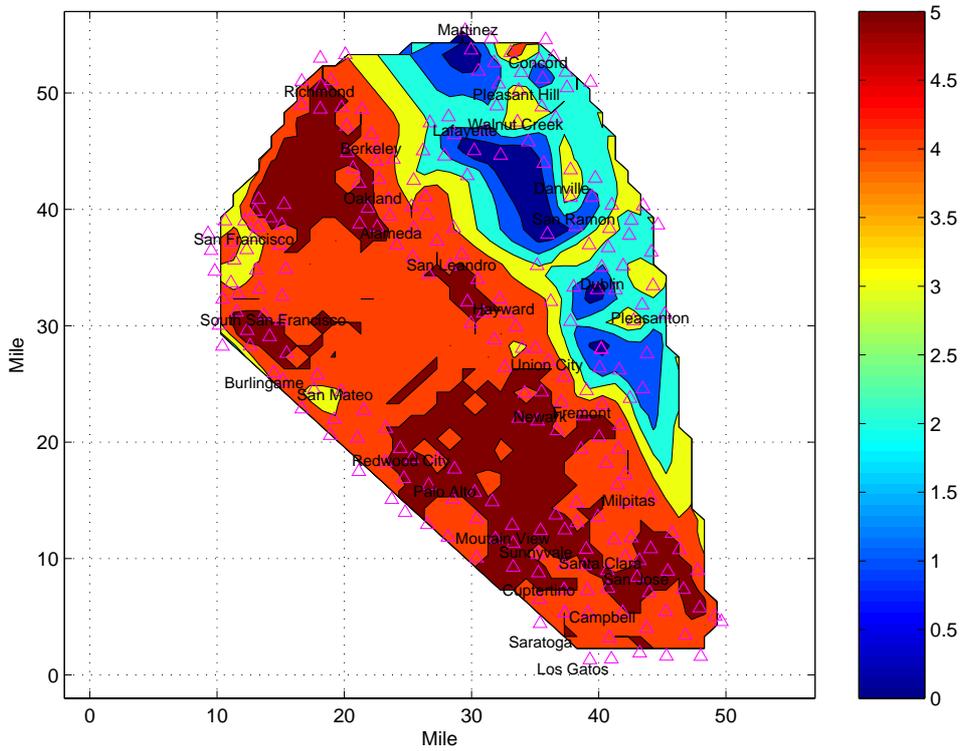


Figure 6: Channel usage map of channels 14-20 (total 7) at the threshold SNR of 8dB.

otherwise). Figure 6 through 8 plot the channel usage maps of the three channel sets assuming a threshold SNR of 8dB. The maximum usages of the three channel sets are about 5/7, 20/30, 9/18 respectively. Comparing those figures with the topographic view of the region in Figure 1, we observe that the maximum usages appear to be consistent over the (low-altitude) areas around the Bay. The inland areas have significantly more spectrum opportunities due to low usages.

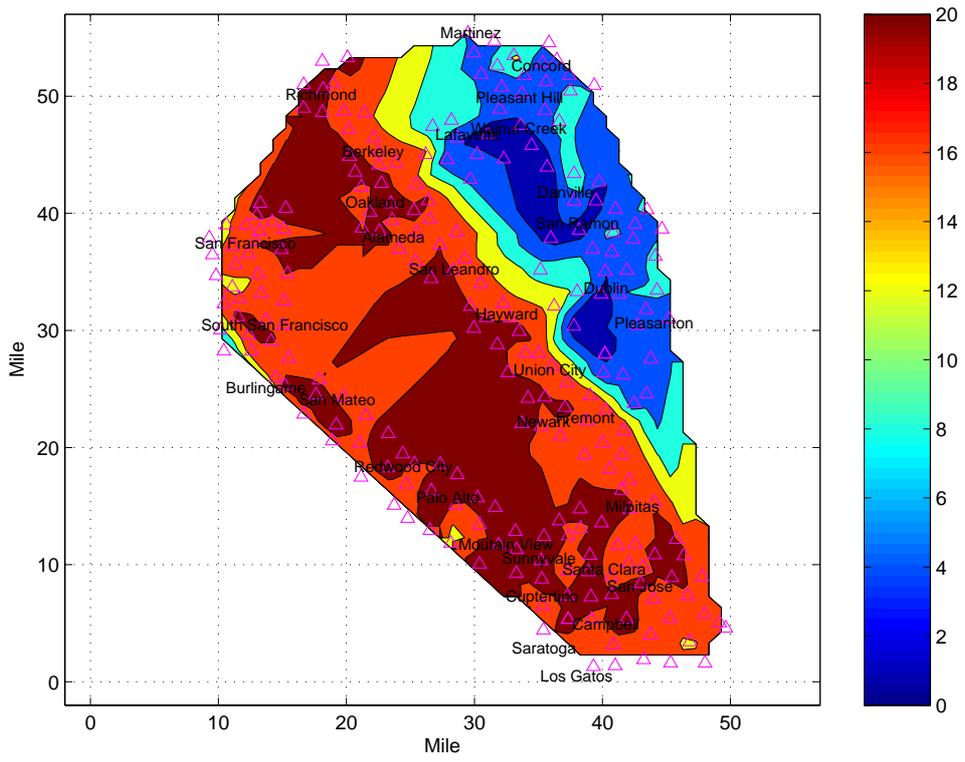


Figure 7: Channel usage map of channels 21-36, 38-51 (total 30) at the threshold SNR of 8dB.

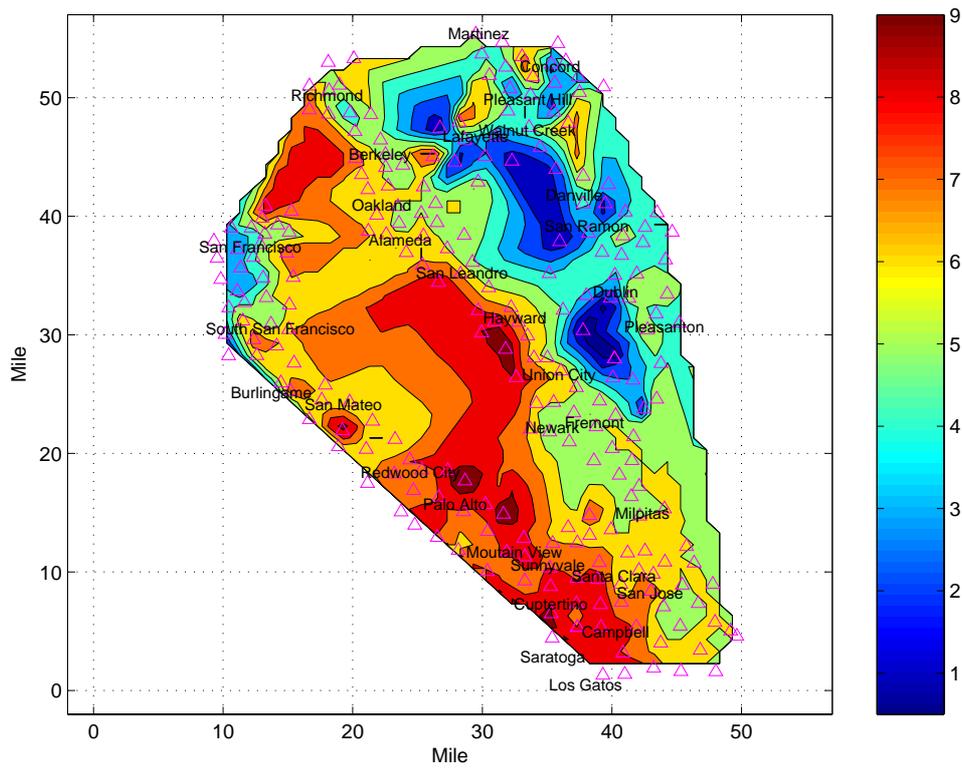


Figure 8: Channel usage map of channels 52-69 (total 18) at the threshold SNR of 8dB.

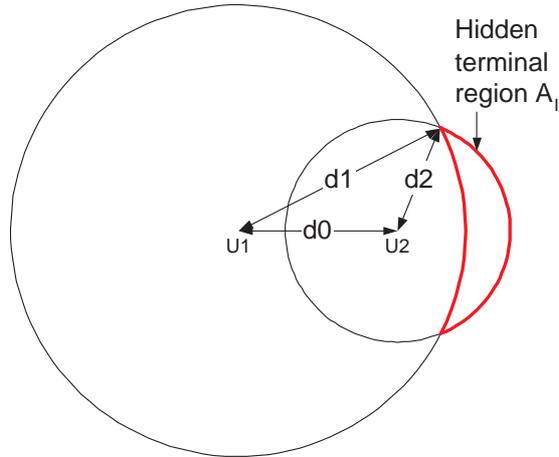


Figure 9: Illustration of hidden terminal problem.

3 Hidden terminal probability analysis

A number of earlier comments on the cognitive radio NPRM have expressed concerns about the potential interference due to hidden terminals. The current NPRM is seeking comments regarding hidden terminal problem and interference avoidance mechanisms.

To systematically address the problem, we first quantify the hidden terminal problem in terms of hidden terminal probability (HTP). We then propose a mechanism to reduce the hidden terminal probability through sensing information sharing among cognitive users which we call *collective sensing*.

We will base our discussion first on a uniform propagation loss model where the signal propagation loss increases monotonically with distance. We then consider the case of shadowing where the loss not only depends on propagation distance but also on environment attenuation.

3.1 HTP with no shadowing

Consider the signal transmission from a primary user U_1 to another primary user U_2 in a propagation environment with uniform loss as illustrated in Figure 9. Let d_0 be the maximum distance within which a primary user can hear another primary user. Let d_1 be the maximum distance within which a cognitive user can hear a primary user. Let d_2 be the maximum distance within which the transmission from a cognitive user is considered *harmful* to a primary user.

As depicted in the graph, for the signal transmission from U_1 to U_2 , any cognitive user who appears in the marked hidden terminal region cannot hear U_1 and will create harmful interference to U_2 if it choose to transmit. Let the area of the hidden terminal region be A_I ,

the worst case interference, i.e. the biggest A_I , happens when U_2 is at maximum distance d_0 from U_1 .

Assuming a cognitive user area density of ρ , the hidden terminal probability – the probability that at least one cognitive user appearing in the interference region A_I – is

$$P_{\text{HT}} = 1 - e^{-\rho A_I} \quad (1)$$

as derived in Appendix A.1. Referring to Figure 9, in order to completely eliminate the interference, we must have

$$d_1 \geq d_0 + d_2 \quad (2)$$

in which case $A_I = 0$ and so is P_{HT} .

Consider the log-distance path loss model [5] where the loss from a transmitter to a receiver at a distance r away is express as

$$\bar{L}(r) = K(r_0) \left(\frac{r}{r_0} \right)^\alpha \quad (3)$$

Here α is the *path loss exponent* characteristic of the propagation environment; r_0 is the distance from the transmitter to a close-in reference point; and $K(r_0)$ is the loss from the transmitter to the reference point. For simplicity, we will assume both the primary user and cognitive user have the same r_0 and $K(r_0)$ (noting that by doing so we effectively assume a TV station transmitter has the same elevation as a cognitive user transmitter and thus significantly overestimates the TV signal path loss.)

As illustrated in Figure 10, suppose the primary user transmission power is P_0 ; the cognitive user transmission power is P_1 ; primary user decoding threshold is P_D ; cognitive user sensing threshold is P_S ; and the harmful interference threshold level is P_H . Since

$$\bar{L}(d_0) = P_0/P_D \quad (4)$$

$$\bar{L}(d_1) = P_0/P_S \quad (5)$$

$$\bar{L}(d_2) = P_1/P_H \quad (6)$$

The interference free condition (2) can be restated as

$$\begin{aligned} r_0 \left[\frac{1}{K(r_0)} \frac{P_0}{P_S} \right]^{\frac{1}{\alpha}} &\geq r_0 \left[\frac{1}{K(r_0)} \frac{P_0}{P_D} \right]^{\frac{1}{\alpha}} + r_0 \left[\frac{1}{K(r_0)} \frac{P_1}{P_H} \right]^{\frac{1}{\alpha}} \\ \Rightarrow \left(\frac{P_D}{P_S} \right)^{\frac{1}{\alpha}} &\geq 1 + \frac{\left(\frac{P_D}{P_H} \right)^{\frac{1}{\alpha}}}{\left(\frac{P_0}{P_1} \right)^{\frac{1}{\alpha}}} \end{aligned} \quad (7)$$

Note that P_D/P_S measures the cognitive user sensing performance against the primary user decoding performance which we call *sensing gain*. P_0/P_1 measures the primary user and cognitive user transmission power difference. Figure 11 plots the required P_D/P_S versus P_0/P_1 to achieve interference-free condition (7) assuming $P_D/P_H = 20\text{dB}$.

Referring to Figure 11, if a typical TV station transmission power is 100kW and a cognitive user's maximum transmission power is 1W ($P_0/P_1 = 50\text{dB}$), a 5dB P_D/P_S will ensure interference-free operation under typical path loss exponents.

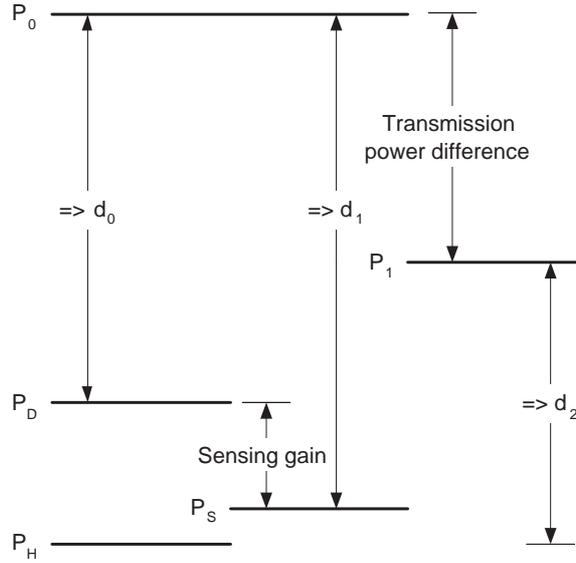


Figure 10: Illustration of relations between various signal power level definitions. P_0 : primary user transmission power; P_1 : cognitive user transmission power; P_D : primary user decoding threshold; P_S : cognitive user sensing threshold; P_H : harmful interference threshold level.

3.2 Improving sensing performance

We separate sensing methods into two categories 1) energy sensing and 2) phase sensing. For example, spectrum analyzer is an energy sensing device. Phase sensing gives better performance but it requires that the target signal contains a known pattern and it is usually achieved by correlating the received signal with the known pattern. The length of the known correlation pattern (in terms of symbol period) determines how much sensing gain can be achieved over the nominal symbol SNR.

Phase sensing can be achieved in TV band cognitive system. For example, the DTV signal contains a Data Field Sync [2] segment $77.3\mu\text{s}$ long every 24.2ms , where most of the data are known. The sensing gain achieved through correlating one Data Field Sync segment is

$$\frac{77.3}{0.186} \approx 26\text{dB}$$

where 0.186 is the symbol period in μs . The NTSC signal contains a narrowband video carrier. If we correlate the signal with a carrier of the same frequency over a long enough period (equivalent to narrowband filtering), we can achieve an arbitrarily good sensing gain.

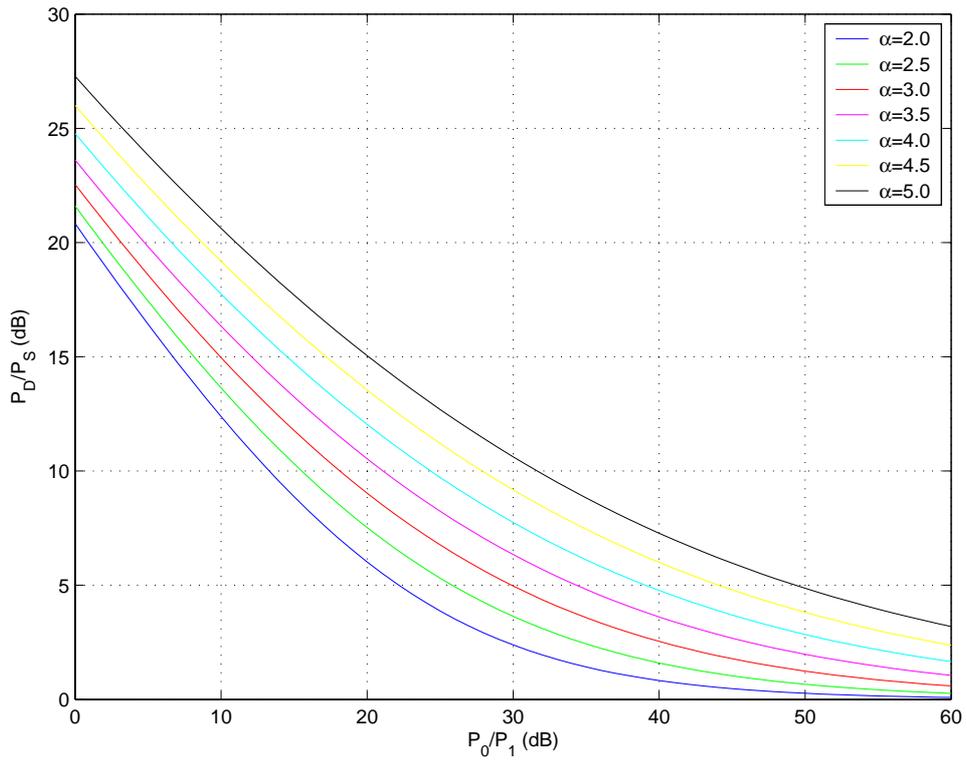


Figure 11: Required P_D/P_S versus P_0/P_1 to ensure interference-free operation in uniform propagation loss environment with different path loss exponents.

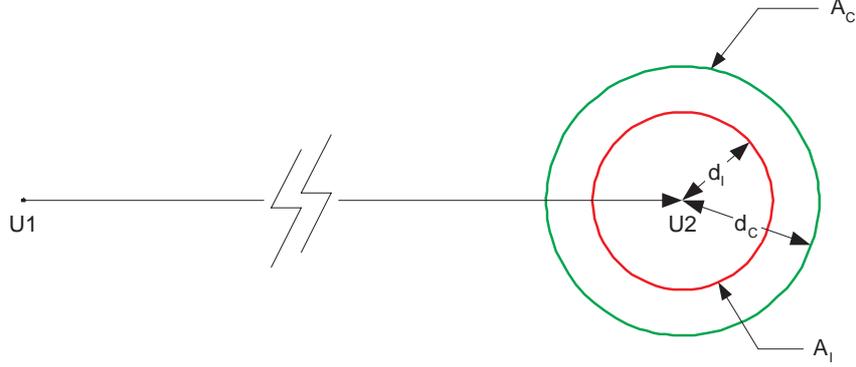


Figure 12: Illustration of hidden terminal problem in shadowing environment.

3.3 HTP in shadowing environment

Realistic propagation environment is non-uniform and receivers at same distance from the transmitter may receive different signal strengths depending on the signal paths between the transmitter and the receivers. This effect is called shadowing. If we write the log-distance path loss (3) in dB form

$$\bar{L}(r) = K(r_0) - 10\alpha \log_{10} r_0 + 10\alpha \log_{10} r$$

the actually loss at a particular receiver is modeled as

$$L(r) = \bar{L}(r) + X_\sigma \quad (8)$$

where $\bar{L}(r)$ is considered as the average path loss while the zero-mean Gaussian random variable X_σ with standard deviation σ accounts for the effect of shadowing [5].

We again consider the signal transmission from primary user U_1 to primary user U_2 as illustrated in Figure 12. If the distance between U_1 and U_2 is r , the distance between a potential interfering cognitive user and U_2 is Δ , and the distance between U_1 and the cognitive user is r' , because the transmission power difference between U_1 and the cognitive user,

$$r \gg \Delta$$

and we have

$$\bar{L}(r) \approx \bar{L}(r + \Delta) \geq \bar{L}(r') \geq \bar{L}(r) \Rightarrow \bar{L}(r') \approx \bar{L}(r)$$

In other words, any potential interfering cognitive user sees the same pass loss for signal coming from U_1 as does U_2 . We consider the case where U_2 is at cell edge, i.e. $r = d_0$. The signal path loss from U_1 to a cognitive user is then

$$\bar{L}(d_0) + X_\sigma$$

The successful sensing of the signal by the cognitive user requires

$$P_0 - [\bar{L}(d_0) + X_\sigma] > P_S \Rightarrow X_\sigma < P_D - P_S$$

and the sensing success probability is

$$\Gamma = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{P_D - P_S} e^{-\frac{x^2}{2\sigma^2}} dx \quad (9)$$

Note that both P_D and P_S are in dB units and Γ is a probability value between 0 and 1.

Because of shadowing, the interference region now extends beyond d_2 with interference probability decreases with the increasing distance from U_2 . For simplicity, we will assume an *effective* interference region around U_2

$$A_I = \pi d_I^2$$

noting d_I should be on the same order as d_2 . Any cognitive user in the effective interference region A_I who has not successfully sensed the signal from U_1 is a hidden terminal.

Given cognitive user density ρ , sensing success probability Γ , and effective interference region A_I , the hidden terminal probability in the case of shadowing is

$$P_{\text{HT}} = 1 - e^{-\rho A_I (1-\Gamma)} \quad (10)$$

as derived in Appendix A.2. Since now $A_I = \pi d_I^2 \sim \pi d_2^2$ is a constant, the only way to reach zero HTP is to have $\Gamma = 1$ which would require infinite sensing gain P_D/P_S . When P_D/P_S is finite, the HTP in Equation (10) is monotonic with cognitive user density ρ . With the increase of ρ , the HTP will eventually reach an intolerable level.

3.4 HTP with sensing information sharing

The above dilemma is a direct result of sensing being performed by a local cognitive user alone. If the user happens to experience severe shadowing, it may not be able to detect the primary user signal even with good sensing gain. When the cognitive user density grows, such situation will happen more often resulting in increased interference to the primary user.

We propose to solve this problem by sensing information sharing between cognitive users. When increasing number of cognitive users share their sensing results, the probability of shadowing reduces exponentially and so does the hidden terminal probability. We call this approach *collective sensing by cognitive user network*.

Referring to Figure 12, suppose all cognitive users in an area

$$A_C = \pi d_C^2$$

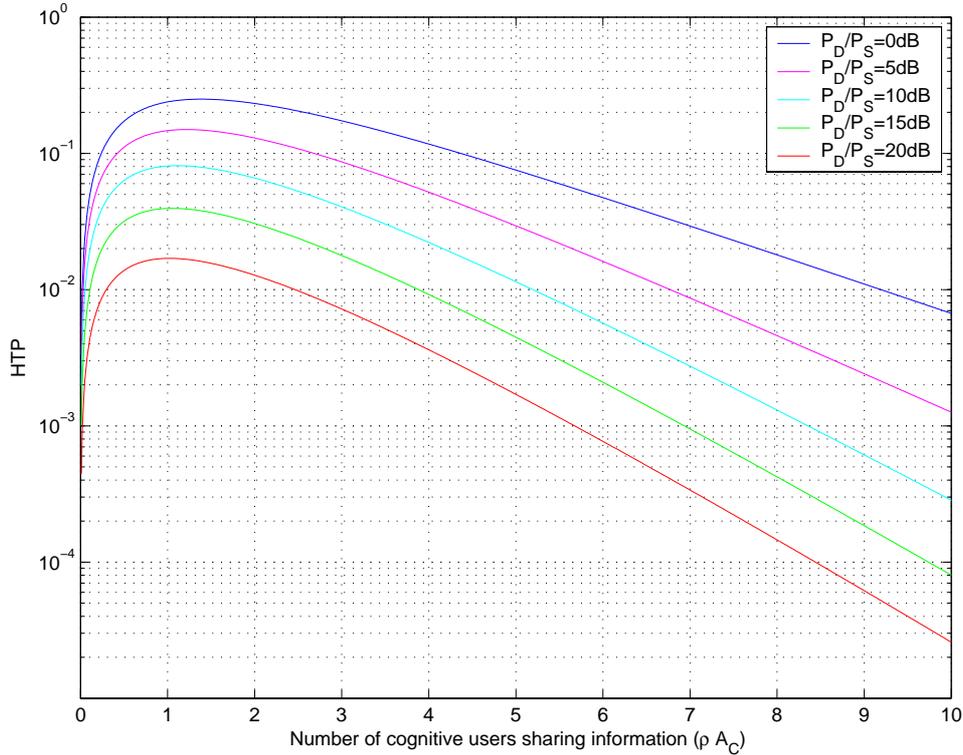


Figure 13: Hidden terminal probability with sensing information sharing between cognitive users in a shadowing environment with $\sigma = 11.8\text{dB}$.

around U_2 share their sensing information so that a cognitive user in A_I only transmits when none of the cognitive users in A_C senses the signal from U_1 . We will assume $A_C \geq A_I$. The hidden terminal probability becomes

$$P_{\text{HT}} = e^{-\rho A_C \Gamma} \left[1 - e^{-\rho A_I (1-\Gamma)} \right] \quad (11)$$

which is derived in Appendix A.3.

Comparing Equation (11) with Equation (10), the added factor $e^{-\rho A_C \Gamma}$ – a result of sensing information sharing – helps drive down the HTP when ρ increases. This effect is plotted in Figure 13 where we assume $A_I = A_C$; Γ is calculated using Equation (9); shadowing environment is characterized by the random Gaussian variable X_σ with $\sigma = 11.8\text{dB}$ from an urban cellular measurement [5].

Figure 13 shows that an arbitrarily low HTP can be achieved with enough number of cognitive users ρA_C sharing their sensing information. And to achieve a target HTP requires at least a threshold number of such cognitive user participants. We call this threshold the *critical mass of collective sensing*. Mathematically, we define the critical mass as

$$M_C = \rho A_C \quad (12)$$

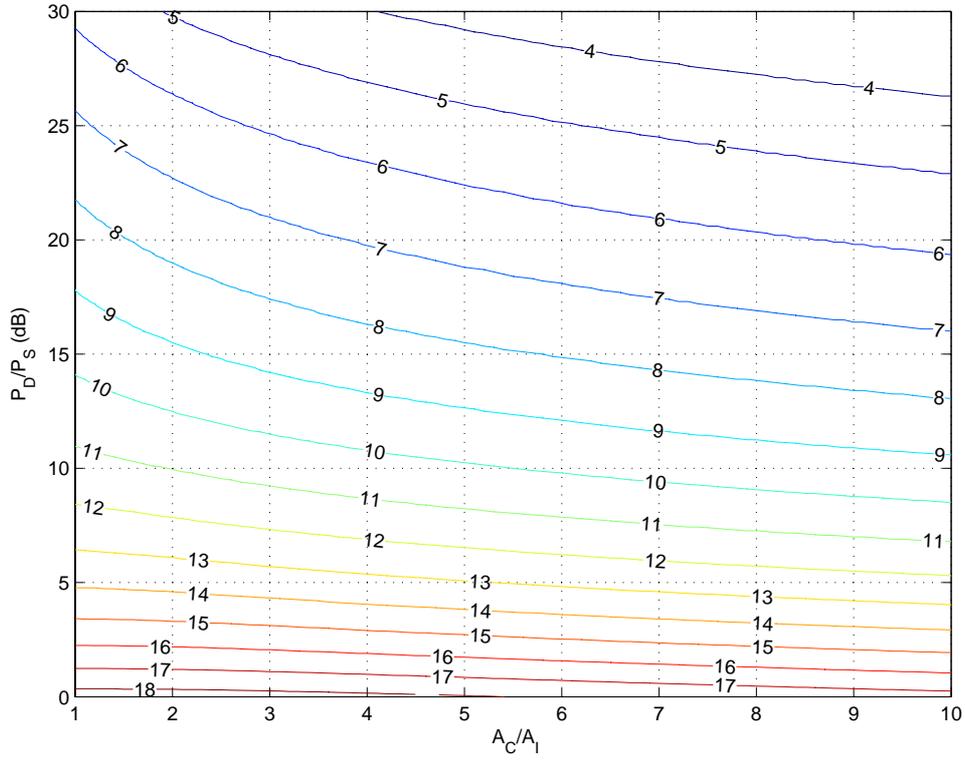


Figure 14: Critical mass of collective sensing to achieve 10^{-4} HTP in a shadowing environment with $\sigma = 11.8\text{dB}$.

such that Equation (11) achieves certain predefined HTP. Figure 14 plots the M_C s to achieve an HTP of 10^{-4} in a shadowing environment with $\sigma = 11.8\text{dB}$ under various sensing gains and area ratios A_C/A_I . For example, when $A_C/A_I = 2$ and $P_D/P_S = 10\text{dB}$, a critical mass of 11 cognitive users are required to achieve the HTP.

4 Conclusion and recommendations

The following is a list of our observations:

- TV signal coverage has significant terrain dependence and coverage contours are irregular.
- We observe on average, 20MHz vacant bandwidth in channels 14-20, 80MHz in channels 21-36 and 38-51, and 70MHz in channels 52-69, in our Bay Area survey.
- Spectrum opportunities in different areas have big disparities. While in some area the channel usage is heavy, in some area the usage is close to zero.
- Hidden terminal problem can be eliminated in a uniform propagation loss environment and with the current NPRM specifications, a 5dB sensing gain would be enough to achieve this goal.
- Sensing performance can be improved using longer signal correlation pattern and sensing gain above 20dB can be achieved in TV band cognitive systems.
- Hidden terminal problem always exists in shadowing environment. If sensing is performed by a local user alone, the hidden terminal problem worsens with increasing cognitive user density.
- Collective sensing (or sensing information sharing) is the key to alleviate hidden terminal problem in shadowing environment and the hidden terminal probability can be reduced to an arbitrary low level with a large number of cognitive users sharing their sensing information.
- With any given set of system parameters, collective sensing requires a critical mass of participants in order to achieve certain hidden terminal probability.

We also have a number of recommendations. First and foremost, as a guiding principle, we believe the Commission should specify rules from the stand point of incumbent protection rather than trying to impose any form of implementation. This would allow greater technology flexibility into the future.

Specifically, we believe sensing based approach should not be eliminated as an option. If the hidden terminal problem was a concern, we have proven in the analysis that through collective sensing it possible to reduce the hidden terminal probability to an arbitrary low level. Comparing to the control signal approach, the sensing approach is more adaptive, scalable, and can better exploit spectrum opportunities.

In addition, the proposed rules section of the NPRM specifies that the cognitive user signal transmission “shall be confined to one or more contiguous television broadcast channels.” We believe limiting the cognitive operation to contiguous channels is unnecessary.

Often times contiguous vacant channels may not exist, in which case the cognitive user would have to operate on a single 6MHz TV channel according to the rule, even if it may actually need larger bandwidth. Technology like wideband OFDM can perform spectrum hole filling over discontinuous spectrum segments and adjacent channel leakage can be controlled through proper windowing and using enough guard bands.

5 Acknowledgements

The work conducted here is supported by Air Force SBIR contract FA8750-04-C-0145. Specially, we would like to thank Robert Husnay from Air Force Rome Research Laboratory for his continuing support in the project. The measurements were conducted in collaboration with Berkeley Wireless Research Center [6] and we would also like to thank their support.

Respectfully submitted,

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A Hidden terminal probability derivations

A.1 Hidden terminal probability

Assume a large number N_∞ of cognitive users are randomly distributed over a large area A_∞ covering the area of interest A . The probability that no user appearing in A , i.e. all users appearing outside A , is

$$\begin{aligned} P_0(A) &= \left(1 - \frac{A}{A_\infty}\right)^{N_\infty} \\ &= \left[\left(1 - \frac{A}{A_\infty}\right)^{\frac{A_\infty}{A}}\right]^{\frac{N_\infty}{A_\infty}A} \\ &= e^{-\rho A} \end{aligned}$$

where

$$\rho = \frac{N_\infty}{A_\infty}$$

is the cognitive user area density. The probability that one or more cognitive users appearing in A is

$$P_{1+}(A) = 1 - P_0(A) = 1 - e^{-\rho A}$$

The probability of k users appearing in A can be calculated as

$$\begin{aligned} P_k(A) &= \binom{N_\infty}{k} \left(\frac{A}{A_\infty}\right)^k \left(1 - \frac{A}{A_\infty}\right)^{N_\infty - k} \\ &= \frac{A^k}{k!} \underbrace{\left[\frac{N_\infty}{A_\infty} \frac{N_\infty - 1}{A_\infty} \dots \frac{N_\infty - (k-1)}{A_\infty}\right]}_{\rightarrow \rho^k} \left(1 - \frac{A}{A_\infty}\right)^{N_\infty} \underbrace{\left(1 - \frac{A}{A_\infty}\right)^{-k}}_{\rightarrow 1} \\ &= \frac{(\rho A)^k}{k!} e^{-\rho A} \end{aligned}$$

A.2 HTP in shadowing environment

The probability that k users appear in the effective interference region A_I is

$$\frac{(\rho A_I)^k}{k!} e^{-\rho A_I}$$

The probability that all k users have successfully sensed the signal from U_1 is

$$\Gamma^k$$

The probability that at least one user has not successfully sensed the signal from U_1 is

$$1 - \Gamma^k$$

which is the interference probability given k users are inside the interference region A_I . The overall HTP is then

$$\begin{aligned} & \sum_{k=1}^{\infty} \left[\frac{(\rho A_I)^k}{k!} e^{-\rho A_I} \right] (1 - \Gamma^k) \\ &= e^{-\rho A_I} \left[\sum_{k=1}^{\infty} \frac{(\rho A_I)^k}{k!} - \frac{(\rho A_I \Gamma)^k}{k!} \right] \\ &= e^{-\rho A_I} \left[e^{\rho A_I} - 1 - (e^{\rho A_I \Gamma} - 1) \right] \\ &= 1 - e^{-\rho A_I(1-\Gamma)} \end{aligned}$$

A.3 HTP with sensing information sharing

For any k users appearing in A_I , the interference only happens when all those k users are not sensing the signal from U_1 plus all users appearing in the area $A_C - A_I$ are not sensing the signal from U_1 . The total HTP is then

$$\begin{aligned} & \sum_{k=1}^{\infty} \frac{(\rho A_I)^k}{k!} e^{-\rho A_I} (1 - \Gamma)^k \left\{ \sum_{l=0}^{\infty} \frac{[\rho (A_C - A_I)]^l}{l!} e^{-\rho (A_C - A_I)} (1 - \Gamma)^l \right\} \\ &= \sum_{k=1}^{\infty} \frac{(\rho A_I)^k}{k!} e^{-\rho A_I} (1 - \Gamma)^k \left\{ e^{-\rho (A_C - A_I)} e^{\rho (A_C - A_I)(1-\Gamma)} \right\} \\ &= e^{-\rho (A_C - A_I)\Gamma} e^{-\rho A_I} \sum_{k=1}^{\infty} \frac{(\rho A_I)^k}{k!} (1 - \Gamma)^k \\ &= e^{-\rho A_C \Gamma} e^{-\rho A_I(1-\Gamma)} \left[e^{\rho A_I(1-\Gamma)} - 1 \right] \\ &= e^{-\rho A_C \Gamma} \left[1 - e^{-\rho A_I(1-\Gamma)} \right] \end{aligned}$$

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