Spectral Efficiency

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Raised Cosine Pulse Shaping

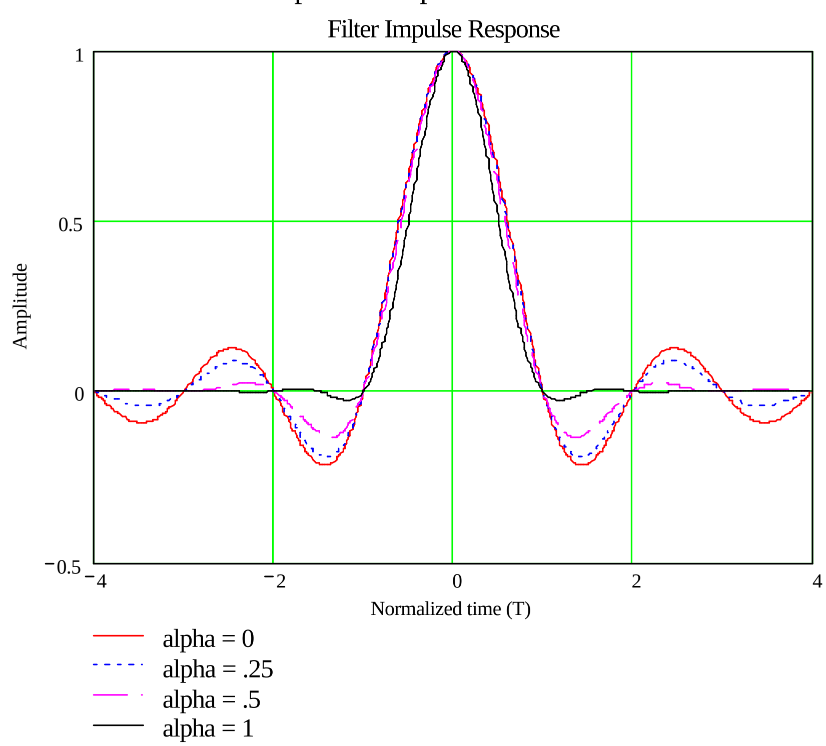
The impulse response corresponding to a raised cosine spectrum is

$$s(t) = \frac{\sin\left(\pi \cdot t/T\right)}{\pi \cdot t/T} \cdot \frac{\cos\left(\alpha \cdot \pi \cdot t/T\right)}{1-\left(2 \cdot \alpha \cdot t/T\right)^2}$$

$$0 \le \alpha \le 1$$

where T is the symbol duration and α is the roll-off factor [1].

The figure below illustrates the impulses responses for several roll-off factors.

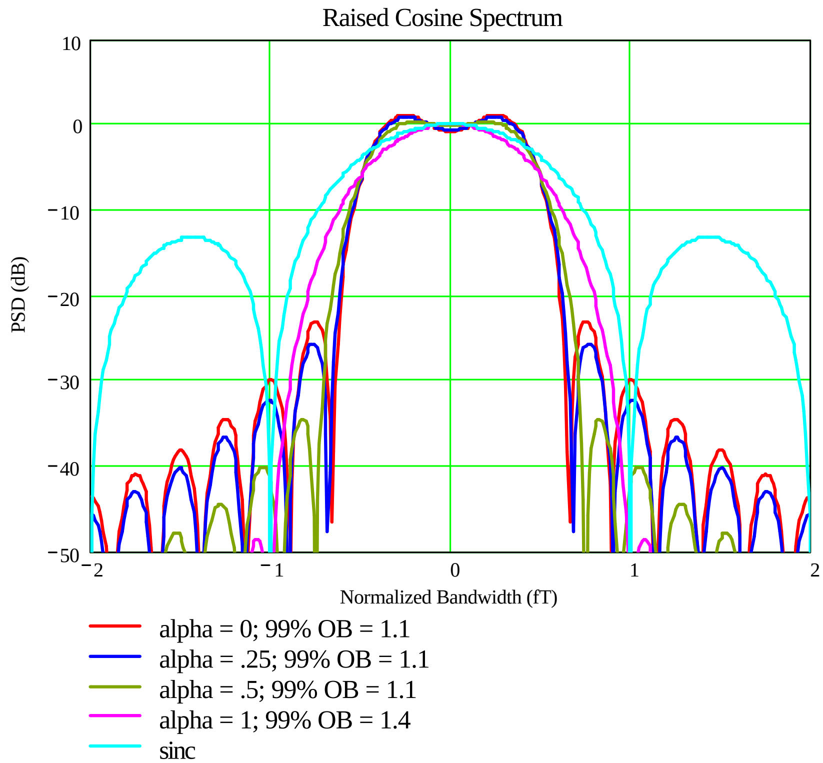


The spectrum of the raised cosine impulse response is a function of the length of the impulse response and the roll-off factor. With an infinite length impulse response, the spectral power is completely contained within a normalized bandwidth ($f\,T$) equal to 1 for $\alpha = 0$ and $f\,T$ equal to 2 for $\alpha = 1$.

However, such a filter is not physically realizable. Therefore we have chosen to restrict the duration of the impulse response to ±2. This will result in a deviation of the spectral shape from an ideal raised cosine spectrum. The truncation of the impulse response causes ripples in the passband and sidelobes. The spectrum of the truncated impulse response is given below.

$$S(f) = \left| \frac{1}{T} \int_{-2}^{2} s(t) e^{-j2pft} \, dt \right|^2$$

The figure below illustrates the spectrum for several roll-off factors. For comparison, the sinc function (spectrum of a rectangular pulse) is also included.



The 99% occupied bandwidth (OB) is calculated by integrating S(f) over a frequency range which equals to 99% of the spectral power, as illustrated in the equation below.

$$\frac{1}{A} \int_{-OB/2}^{OB/2} S(f) \, df = .99$$

$$A = \int_{-\infty}^{\infty} S(f) \, df$$

The OB values for several roll-off factors are given in the figure of the spectrum above.

Spectral Efficiency

We define spectral efficiency as bits/sec/Hz. We expand this as follows

$$
\frac{\text{bits}/\text{sec}}{Hz} = \frac{\left(\text{bits}/\text{symbol}\right)\left(\text{symbol}/\text{sec}\right)}{Hz}
$$

$$
= \frac{M \cdot 1/T}{BW} = \frac{M}{BW \cdot T}
$$

$$
= \frac{M}{OB}
$$

where M is the modulation constellation size (i.e. M = 1 for BPSK or OOK and M = 2 for QPSK).

As described in the previous section, the occupied bandwidth ranges from 1.1 to 1.4. Therefore an uncoded BPSK system will have a spectral efficiency ranging from 0.7 to 0.9. Typically digital communications systems employ channel coding to protect against bit errors. With rate ½ coding, a BPSK system will have a spectral efficiency ranging from 0.35 to 0.45.

Similarly, a QPSK system with rate ½ coding will also have a spectral efficieny below 1. To achieve a spectral efficiency above 1 with channel coding will require complex modulation orders of 3 and above.

Reference

[1] Benedetto, S., et. al., “Digital Transmission Theory”, Prentice-Hall, Inc., 1987.