Spectrum Deregulation Without Confiscation or Giveaways

Michael H. Rothkopf
Rutgers Business School and RUTCOR
Rutgers University
640 Bartholomew Road
Piscataway, NJ 08854-8003
rothkopf@rutcor.rutgers.edu

Coleman Bazelon
Analysis Group/Economics
1747 Pennsylvania Avenue, Suite 250
Washington, DC 20006
cbazelon@analysisgroup.com

Draft of January 9, 2003

Abstract

The right to use US radio spectrum is extremely valuable. Unlike most property rights, licenses to use radio spectrum usually carry significant restrictions on how the radio spectrum is used. Economic efficiency requires an expansion of the existing rights to allow the spectrum to be used for its most valuable use. Distributing the expanded rights to use radio spectrum raises questions of both efficiency and equity. This paper proposes an auction mechanism for distributing additional rights. It overcomes the fact that the holders of existing rights have a strong competitive advantage over any challengers. It also deals with the need to allow combinatorial bids since economic new spectrum uses may require the use of spectrum licensed to more than one current license holder. It does this without crossing the limits imposed by computational complexity. This auction mechanism could be a useful addition to the arsenal of tools available to the FCC.

1. Introduction

In the United States, the radio spectrum is legally the property of the public, but with varying degrees of private use rights. The radio spectrum is extremely valuable—many times more valuable than all of the gold in Fort Knox (See Snider *et al*. 2003). For over three quarters of a century, the government has been making policy with the aim of having this valuable asset used in the public interest—a flexible standard that has evolved over the years. Those interested in a fuller discussion of U.S. spectrum management should consult Congressional Budget Office 1997. At present, some spectrum is held for government uses—defense, air traffic control, public safety, etc.—and some is licensed to companies for a variety of particular uses such as broadcasting and fixed and mobile communications. Almost all spectrum of value with today's technology has been assigned or reserved for some use. Those interested in details on the current spectrum allocations can consult Kobb 2001.

The current system of spectrum management is based on a command and control framework. The Federal Communications Commission (FCC) manages private and state and local government uses of spectrum and the National Telecommunications and Information Administration (NTIA) manages the Federal uses of spectrum. For non-Federal uses, traditionally the FCC allocates blocks of spectrum to types of uses, such as broadcasting or satellite, creates channel assignments and then assigns licenses to users. Licenses often specify where, when and how the licensee may use the radio spectrum. For instance, a typical television license will specify a transmitter tower location, height, power levels, channel assignment and broadcast technology.

Licenses traditionally were distributed on a first come basis. When more than one applicant wanted a particular license, the FCC was forced to choose among applicants. For most of its history it used the comparative hearing, commonly referred to as a beauty contest. This became an expensive and inefficient procedure and was replaced with lotteries in the 1980s. Beginning in 1994, the FCC was given the authority to use auctions to assign licenses that had mutually exclusive applications. The FCC has pioneered innovative auction formats to assign rights to use radio spectrum. The assignments to date have generally been for cases where either there were no significant incumbents or there were clear rules for the incumbents to clear out the band of spectrum in the license.

Starting with Ronald Coase (1959) who built upon the insights of Leo Herzel (1951), some economists have argued that the best way to get the spectrum used efficiently was to assign private property rights to the spectrum, e.g., Hazlett 2001 and many of the references in it. This idea has slowly gained respect. Currently, a broad consensus exists that for spectrum in private use without special market failures, privately owned rights is the most effective way to have the spectrum used most efficiently. There are many uses of spectrum that are not suited to a regime of private property rights. For example, unlicensed spectrum used for cordless phones and garage door openers is more effectively utilized without any private entity owning the spectrum. This paper will not address when spectrum should be under a private property regime. Instead, we take the decision to expand the rights in a given band of spectrum as given and look to how those expanded rights are distributed to private entities.

Many interesting questions are raised in trying to define the scope and nature of the rights that should be attached to radio spectrum. At one extreme are fee simple property rights and at the other extreme are time-limited, royalty-based rights leases. These are important questions, but this paper is agnostic with respect to them. It is concerned with the method of distributing expended rights, whatever they are.

There are at least two problems inherent in distributing expanded property rights in spectrum. First, there is a desire to respect and extend indefinitely the rights granted to current licensees, even when those licensees received their licenses free, for a specific purpose, and for a limited time. Second, both fairness and efficiency require that the government receive most of the value of the liberalization of the licenses. Since the right to use the spectrum is worth hundreds of billions of dollars, the fairness aspect of a spectrum giveaway probably requires little comment. However, the efficiency argument is more subtle, and it is critical since the case for "privatizing" the spectrum is based upon efficiency. The essence of the argument is that if the government fails to get full value for assets it gives away, the money it does not receive must be raised with taxes.[1] (See Rothkopf and Harstad 1990 for a discussion.) There is a substantial economic literature documenting the marginal inefficiencies associated with raising money from income taxes.[2] See Ballard *et al*. 1985, Fullerton 1988, and Stewart 1984 for details. A conservative estimate is that for every three dollars in revenue forgone (and, therefore in additional taxes raised) there is a dollar of lost productivity.

A fairly broad consensus has formed on the answer to the first of these two problems—that incumbents should be fully protected in the rights they currently enjoy. The reasons for the desire to protect incumbents vary widely, from concerns for "fairness" to licensees who were not the original holders of the license, to concerns about incentives for investments needed to fully exploit licenses, to political judgments about licensees' abilities to block reforms.

Given this consensus, dealing with the second problem presents some difficulties. It is hard to have a competitive sale of the rights to liberalize the use of a block of spectrum that already has been licensed for a specific use. (We will call these rights "relaxation rights.") It is a bit like trying to sell a valuable block of downtown real estate when someone has the right to have a lemonade stand on it. Who will offer to pay anything near its real value when the owner of the rights to the lemonade stand can block any potential use of the property. (This example is not far fetched. The right to broadcast television on a UHF station in a major city where almost everyone who watches the station gets their signal over cable is probably worth a few percent of what the spectrum would be worth for mobile communications. See Kwerel and Williams, 1992 and Snider *et al*. 2003.) Normally, if such downtown real estate were put up for competitive sale, the owner of the lemonade stand rights or someone in partnership with him would be the only serious bidder. With only one bidder, market forces could not be relied upon to set a price that comes anywhere close to the value of what is being sold. The purpose of this paper is to propose a way to overcome this difficulty.

We describe an auction procedure that can be used to sell relaxation rights that liberalize the use of spectrum while obtaining for the government the fair value of the licenses it is granting. The heart of the proposal is an adaptation of a procedure suggested by C. Bart McGuire and used in the early 1980s by the U.S Department of the Interior to auction coal rights to Federal coal tracts where there were owners of adjacent coal deposits who were the only logical bidders. (See U.S. Department of the Interior, 1981.) In the context of coal, the approach was called "intertract competition." It made the bidders for different coal tracts compete with each other. This approach was authorized by Congress and evaluated favorably by the "Linowes Commission" (Commission on Fair Market Value Policy for Coal Leasing, 1984 at pp. 216-221) set up by Congress to investigate the handling of coal leasing by President Reagan's Interior Secretary Watts that lead to a shut down of Federal coal leasing.

[1] We are assuming that Congressional budget enforcement rules are not gamed in the implementation of the enabling legislation for the auctions proposed in this paper.

[2] While there may be unused opportunities to tax pollution or other externalities, these are likely to be relatively small, and the marginal source of tax revenue is the income tax.

The proposal also draws on two other ideas. One is the idea of treating a constraint on the total amount to be sold as "soft." This idea dates back to discussions of "PURPA auctions" for electricity supply contracts. (See Rothkopf *et al*., 1990.) The combinatorial bid version of the proposal also draws on the idea of rejecting bids when their "inflexibility" would constrain the solution to the bid selection problem. This idea is used in the auction of natural gas pipeline capacity by the Natural Gas Company of America, and was proposed by them.

Under this proposal, no licensee's rights will be damaged or limited in any way. However, under this proposal, no licensee or other party will get spectrum license rights without competition. In particular, those with rights for a use that greatly under uses spectrum will have to compete with others to get their license restrictions eased even though they may be the only bidder for the particular rights that compliment theirs. **It is not necessary to give away spectrum rights in order to have the advantages of private ownership incentives**.[3]

The next section of this paper presents the proposal, first in simplified form and then in a more complicated form in which bidders can make offers on relaxation rights on combinations of licenses. This is followed by a discussion of the proposal, of implementation issues, and of some specific concerns in spectrum management policy such as public interest obligations.

2. A Simplified Proposal.

Here is a simple version of the proposal to expand spectrum license rights without either giving it away for much less than its value or forcing the holders of existing rights to release them.

Under this simplified proposal, Congress will authorize the FCC to announce an annual series of auctions of "overlay" spectrum rights. Each auction will release a given amount of spectrum (measured in units of bandwidth times the population area covered, i.e., in MHz-Pops[4]) for essentially unrestricted use subject to responsibility for noninterference with licenses for other frequencies and other geographic areas. However, the amount to be sold in a single sale will be a small fraction, perhaps 10%, of the amount upon which bids will be accepted. While for national security, public safety, or other special purposes some spectrum may be excluded from bidding in these sales, relaxation rights for most privately licensed spectrum will be eligible for sale and sold if the offer for it is high enough. Any currently licensed spectrum will be offered subject to the rights of the current spectrum holder. The current license holder may bid to relax the restriction on her license. Others may also bid for these relaxation rights although other bidders may well be at a disadvantage relative the current rights holder. (Essentially unrestricted rights can be offered for unlicensed spectrum.) The auction will be a sealed-bid, market-clearing-price auction. In this simple version of the auction, there will be no combinatorial bids and spectrum with the highest bids per MHz-Pop[5] will be sold up to

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[3] Cf. Rossten *et al*., at page 6: "Efforts to extract gains from licensees … should not be permitted unduly to hinder or delay realization of the public benefits from promoting greater competitiveness through spectrum liberalization."

[4] The units here may be a bit confusing to some. Dollars per MHz per Pop is the same as dollars per MHz-Pop. However, MHz-Pops, which are appropriate here, differ from MHz per Pop.

[5] Not all MHz-Pops are the same. For example, lower frequencies that are suitable for mobile communications are more valuable than the upper frequencies. It is important for the auction that bids be

the cut off limit on MHz-Pops for the sale. The important consequence of this is that a license holder wishing to relax the constraints on a license will have to compete for the right to do so with holders of other licenses who also wish to relax the constraints on their licenses. The auction can be improved in some ways by adding some complications such as allowing bidders to submit combinatorial bids, and we will consider this below.

In this simple version of the auction, in order to select the winning bids the FCC will first rank order the bids with respect to the amount offered per MHz-Pop. Starting with the highest ranked bid, the FCC will award eligible bids that do not conflict with previously accepted bids until it reaches a bid that would put the total sold over the limit set in advance for the auction for MHz-Pops. This bid is the marginal bid and will set the price per MHz-Pop for all accepted bids (whether it itself is accepted or not). If accepting the marginal bid would make the total MHz-Pops sold exceed the announced target by less than a pre-announced tolerance percentage, the bid will be accepted. If accepting the bid would result in exceeding this tolerance limit, then the FCC will reject the bid. All bids offering a price per MHz-pop less than the marginal bid will be rejected. If the bid acceptance process ends without reaching the target number of MHz-Pops, then all bids that have not been rejected will be accepted and the price per MHz-Pop will be the minimum allowable bid.

Two easy to show theoretical results are worth noting. First, if the tolerance limit exceeds the size in MHz-Pops of the largest license, then the auction will always end with the acceptance of the marginal bid. Second, whenever the auction ends with the acceptance of the marginal bid, the total value expressed in the accepted bids is the maximum possible for the number of MHz-Pops sold.

In auction design, the devil is in the details. It is vital that a number of procedural details be set up correctly. Substantial deposits should be required of bidders, and there should be prompt payment by winners and prompt awards to them upon completion of the auction. If citizenship or other qualifications are required, bidders should be required to assert under oath at the time the deposit is made that they meet them. All eligibility challenges except ones connected with criminal prosecutions for perjury should be limited to the period before the auction.[6]

Immediately after the auction, the FCC should return deposits on unsuccessful bids. Successful bidders will pay the remainder of the price of what they have won, and licenses will be awarded to them. If they fail to pay, they will be in default, should lose their deposits and get no rights, bankruptcy laws notwithstanding.

Before each annual auction, the FCC will announce to potential bidders the geographic units (and their populations) that will be used and any frequencies that are not available. If some frequencies are available in some geographic regions but not others, this too will be announced. For simplicity, we will call the units the FCC announces "licenses" even if in some cases part or all of what they cover is currently unlicensed. All frequencies not explicitly excluded will be available subject to the specific rights of existing licensees. The FCC will also announce the tentative target total number of MHz-

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on the same basis so that they can be meaningfully compared. Consequently, lower band and upper band relaxation rights should be sold in separate auctions. Further refinements in the $/MHz-Pops based on the frequency of the band in the bid *may* be considered useful.

[6] The purpose of this proposed procedure is to prevent competitors of the service to be offered by the new licenses from delaying their competition.

Pops to be sold. Bidders should not be greatly surprised by the announcement since a long-term plan for making frequencies available will have been adopted. The FCC will also announce the deposit required per MHz-Pop that the bidder will be eligible to bid on. The deposit will be a substantial fraction of the anticipated price per MHz-Pop in the sale. It may also serve as the minimum bid per MHz-Pop which should also be a substantial fraction of the anticipated price. In order to avoid a noncompetitive auction, after the deposits are received the FCC will, if necessary, announce a reduced total of MHz-Pops to be sold so that the amount to be sold is no more than some pre-announced fraction, say 1/4, of the total eligibility.

Note that this is a one-time, sealed-bid auction. Hence, there is no need to restrict the bids to round numbers to prevent signaling. Since the bidders have the possibility and incentive to use lots of significant digits in their bids, ties should be exceedingly rare. If ties become common, collusion should be suspected. To discourage tacit collusion, bids at the exact same price should be treated as a single bid. If accepting this "bid" would result in too much spectrum being sold, all of "it" should be rejected.

An Example

Before going further, it may be useful to work through a small example. Table 1 gives the highest nine of a large set of bids. For convenience, the bids have been numbered in decreasing order of bid amount

Table 1. A Set of Bids

Bid #	$/MHz-Pop	License #	MHx-Pops ($x10^6$)
1	6.0121	4321	60.2
2	5.8327	5432	43.5
3	5.7511	4321	60.2
4	5.6330	6543	12.7
5	5.5112	7654	44.0
6	5.5081	8765	32.6
7	5.0423	9876	25.8
8	4.8899	1234	10.4
9	4.8001	2345	10.9
etc.			

Suppose that the government has announced that it will sell relaxation rights for 200 ($x10^6$) MHz-Pops with a tolerance of 5%. In this case, it will accept bid 1 and bid 2. It will reject bid 3 because it has already sold the relaxation rights to license # 4321 to bid 1. It will then accept bids 4, 5, and 6. This brings the total MHz-Pops of accepted bids to 183.0 ($x10^6$). Bid 7, if accepted, would bring the cumulative number of MHz-Pops of accepted bids to 208.8 ($x10^6$). Since this is within 5% of the target of 200 ($x10^6$), the bid will be accepted and its price will set the price of all accepted bids at $5.0423 per MHz-Pop. If the tolerance were only 2.5%, bid 7 would cover too many MHz-Pops to accept. It would be rejected, but it would still set the price for all accepted bids. Bids 8 and 9 would not be accepted even though bid accepting 8 would leave the total MHz-Pops sold below 200 ($x10^6$) MHz-Pops and accepting bids 8 and 9 would leave the total at 203.3 ($x10^6$) MHz-Pops, below 102.5% of that amount.

In this example, accepting bid 8 or bids 8 and 9 after rejecting bid 7 would create two anomalies. First, a bid has been rejected that would have offered a higher unit price than an accepted bid. Second, a bid below the demand curve would have been allowed to set the price. This second anomaly could be quite large, if for example, there was a very low bid for relaxation rights on a license with just 1 ($x10^6$) MHz-Pops. It could be accepted in addition to bids 8 and 9 and still leave the total of accepted bids below 205 ($x10^6$) MHz-Pops.

3. Allowing Combinatorial Bids

It is quite possible that the relaxation rights on an FCC license are worth more if the relaxation rights on other licenses are also obtained. This effect could be mild or it could be critical as when a proposed communication service would absolutely require the relaxation rights of many existing licenses. In addition, it is possible that relaxation rights on alternative licenses would allow a proposed service so that a bidder would like to offer bids in the alternative. Finally, it is possible that bidders are capital limited and would like to limit their total expenditures in an auction. Thus, it is potentially quite useful to allow bidders to bid for combinations of relaxation rights rather than just for individual rights and subject to constraints on their bids. However, allowing bids on combinations and such constraints makes selecting the winning bids more difficult. Furthermore, it is potentially possible that allowing such restrictions on bids would allow bidders to gain an unfair advantage by winning a license at a lower price than another bidder is willing to pay simply by being willfully inflexible.[7] Auction design choices must deal with these possibilities.

We do not know how to deal "optimally" with these trade offs, but we can propose an auction form that deals reasonably with them. It is better than allowing no bids on combinations, but it will not always be better. The basic idea is simple. Bidders may link bids for different licenses together with a constraint. For example, a bidder may offer $4.00 per MHz-Pop for license 1 and license 2. Because these bids are linked together, they are not separate offers to buy either license 1 or license 2 at $4.00 per MHz-Pop; they are just an offer for the combination. The constraint will always be honored. However, if it is an active constraint--that is, if in its absence the optimal allocation of licenses to bidders would be different--then the bids affected by it will be rejected.

The following example may be instructive. Two items of the same size, A and B are for sale. Bidder 1 values A at 2, B at 2, but the combination of A and B at 8 --4 per unit for each item. In the absence of combinatorial bidding, her only safe course is to offer 2 for A and 2 for B. If she offers more, she risks winning one and not the other and suffering a loss. In the proposed auction, she can offer a bid of 2 for A, a bid of 2 for B and a bid of 4 for A linked to a bid of 4 for B for a total of 8 for the combination of A and B. If the best competitive bids are 3 for A and 3 for B, she will win. However, if the best competitive bids are 1 for A and 6 for B, she will not win because the constraint linking

[7] For example, if a bidder with no value synergies between licenses is allowed to link bids on two licenses together so that both or neither of them must be awarded, then it is possible that another bidder offering a higher price for one of the licenses will lose. The inflexible bidder would then have bested a bidder offering more solely because of her inflexibility. Of course, if as is likely, only one bidder can bid competitively for each license, then this situation will not arise.

her bids for A and B together is binding. It prevents the bid taker from taking her bid of 4 for A while rejecting her linked bid of 4 for B and thus getting 4 for A and 6 for B for a total of 10. Thus, this approach is not ideal in this situation.[8] However, it allows some combinatorial bids to succeed, it prevents the kind of problem described in footnote 5, and, as will be discussed, it also avoids some computational problems associated with allowing unlimited combinatorial bids.

Here is the specific auction we propose. It is a one-shot, sealed bid auction. In it, each bid is a price per MHz-Pop for the relaxation rights to a particular FCC license covering a given geographical area, and its population, and range of frequencies. Each bid is made subject to several possible constraints by the bidder. The first such constraint is the eligibility constraint. Based upon her deposit, each bidder is limited to a given number of MHz-Pops. This does not constrain the number of MHz-Pops that she can bid upon, but it does constrain the number she can win. If her deposit is large enough, this constraint will not be binding.

The second kind of constraint is a budget-like constraint specified by the bidder to apply to the sum of her successful bids (not, however, the market-clearing price that the bidder would have to pay). This constraint prevents her from winning relaxation rights that will, in total, cost more than she is willing to spend. Its use by a bidder is voluntary. Bidders would probably prefer to have a budget constraint on actual expenditures, but this might cause computational difficulties. Since budget constraints are often "soft" constraints, this kind of budget constraint may prove useful. It would allow bidders freedom to bid on many different alternatives. In its absence, bidders might find their managements imposing more drastic "exposure" constraints on their total bids as is common in simultaneous sealed-bid auctions for offshore oil rights. (See Rothkopf 1977.)

The third kind of constraint is one used to link two or more bids together. These are the combinatorial constraints discussed above. Bidders may have many such constraints, each linking two or more bids together. Each bid can only be involved in one such constraint. However, a bidder may make separate bids on the same license. The use of such bids by bidders is voluntary.

The fourth kind of constraint is an " exclusive or" or alternative constraint. A bidder can always make two bids mutually exclusive by including the same license in both. Thus, if a bidder has a pair of linked-together bids for relaxation rights for licenses A and B, and another pair of linked-together bids for relaxation rights for licenses B and C, both pairs cannot win since relaxation rights on license B can only be sold once. Thus, the bids will, in fact, be treated as bids for {A and B} or {B and C}. In addition, the FCC could and should allow each bidder a limited number of "pseudo-items" it can bid on.[9] By including such a pseudo–item in a constraint linking bids together, a bidder can make two sets of bids mutually exclusive even though they don't actually overlap. Thus, for example, a bid for A, B and pseudo-item 1 is in conflict with a bid for C, D and pseudo-item 1. The use of such alternative constraints will allow bidders to attempt two or more independent ways to reach a given goal.

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[8] Of course, *if* the bidder could anticipate the competitive bids, she could tie together offers of 2.5 for A and 5.5 for B for the same total of 8. In this case, the constraint tying her bids together would not be binding. However, bidders may not always be able to anticipate competitive bids, so the caution is justified.

[9] The idea for doing this is due to Fujishima *et al.* 1999.

In addition to these bidder-specific constraints, the selection of winning bids is constrained by the (soft) limit on the number of MHz-Pops to be sold and by the requirement that the relaxation rights on each license either be sold once or not sold at all. We propose that subject to all of these constraints, the FCC select the bids that maximize the value expressed in the bids it accepts.[10] This mathematical problem is stated formally in the Appendix. It is an integer programming problem, which implies that it is in a class of problems that are potentially computationally difficult.[11] However, just as in the simplified problem discussed above, we avoided exact solution of the problem initially faced, we plan to avoid this problem here. As discussed below, we plan, instead, to solve a series of linear programming problems. Unlike integer programming, linear programming problems are not in the potentially computationally difficult class of problems.

One computational concern deserves special mention. With this auction form, bidders may have little to lose by submitting many slightly different conflicting bids. If the FCC anticipates the total number of bids posing a computational problem, it can require a nominal processing charge with each bid. This will not inhibit any serious bids, but could head off computational problems. In addition or instead, the FCC could impose a generous limit on the total number bids a bidder could submit. This would let a bidder express all of her important values.[12]

We will now describe the general computational procedure for selecting the winning bids and setting the market-clearing price for a given set of bids. After we have done this, we will give a simple example.

The solution procedure begins by solving the integer programming problem given in the Appendix as a linear program. That is, the problem of maximizing value reflected in accepted bids is solved ignoring the constraints that force variables to take on integer values. We will call this Step 1. If this calculation happens to find a solution in which all of the variables are, in fact, integers, the solution also solves the integer programming problem and is accepted as the solution to the bid acceptance problem. The lowest unit

[10] An alternative, which we do not endorse, would be for the FCC to select the set of bids that would maximize the revenue it receives. Doing so might lead to the FCC withholding licenses in order to increase the revenue from the sale by preventing the marginal price from falling. We believe that the public will be served best if the FCC makes and sticks to an overall judgment on the best pace at which to release spectrum from regulation taking into account both the efficiency gains from public revenue (which will remove the need for an equivalent amount of taxation) and the ability of industry to finance and make available valuable services to the public. It is also possible that this alternative could affect the incentives of bidders in an undesirable way.

[11] Some mathematical problems have solution algorithms that even in the worst case grow in length as the size of the problem is increased no more than a given polynomial bound. Such problems are usually considered computationally easy. On the other hand, some problems have no known algorithm that is guaranteed for the worst case to grow no more than a polynomial bound as the size of the problem increases. Large instance of these problems are potentially unsolvable. Integer programming is in the latter class of problems. See Rothkopf, Pekec and Harstad 1998 for a discussion of this in the context of combinatorial auctions.

[12] See Park and Rothkopf 2001 for a discussion of the effect of limiting the number of combinations a bidder in an FCC auction can make and for a report of a related experiment.

price (in $ per MHz-Pop) of any accepted bid, where the unit prices of bids tied together are combined,[13] is used to set the unit price of the relaxation rights.

If, as is quite likely at first, some integer constraint is violated, the procedure then goes on to modify the problem. This is Step 2. If the budget constraints of a bidder is binding and this results in the proposed sale to that bidder of a fraction of the relaxation right on a license, that budget constraint is tightened to eliminate the fractional purchases. If more than one bidder is in this situation, all of their budget constraints are tightened so that none of them are buying a fractional part of a relaxation right. Similarly, if the eligibility constraint of one or more bidders is binding and this results in the proposed sale to that bidder of a fraction of a relaxation right on a license, the eligibility constraints of those bidders are tightened to eliminate the fractional purchases. It is appropriate to make all of these changes simultaneously since reducing the fractional purchase of one bidder with a binding budget or eligibility constraint, will not eliminate a fractional purchase by any of the others. The calculation then returns to Step 1.

If no budget or eligibility constraint results in the purchase of a fractional relaxation right, then Step 3 checks whether a linking constraint results in the sale of a right to a bidder who values it less than another bidder. If so, the linked bids are rejected and the calculation returns to Step 1. If not, the constraint on the total number of MHz-Pops to be sold must result in the proposed sale of a fraction of a relaxation right on a license or a set of linked-together licenses, and the calculation proceeds to Step 4.

In Step 4, the calculation checks to see if relaxing the constraint on the total MHz-Pops to be sold, but not beyond the pre-announced tolerance limit, will result in the sale of all of those rights and do so without violating a budget-like constraint or an eligibility constraint. If so, the constraint on the total MHz-Pops to sell is relaxed in order to make that sale. The calculation is then complete, all of the bids in the optimal solution are accepted, and the price per MHz-Pop for all sold rights is set by the price of this marginal bid (including the prices per MHz-Pop of any rights to which it is linked by a constraint). If relaxing the constraint on the total number of MHz-Pops to the tolerance limit results in the violation of the budget-like or eligibility constraint of the bidder who made the offer on the marginal license(s), that offer is eliminated and we return to Step 1. If, with one possible exception, the maximum relaxation still leaves the marginal offer or offers only partially filled, then all of the marginal offers are rejected, but their unit price is used to set the price for all accepted bids.[14] All of the other bids in the optimal solution without the relaxation are accepted.

The exception mentioned above occurs when a bidder's bids on a combination is in conflict with another of its own bids at a higher unit price that includes part of the

[13] To combine separate bids, their unit prices are weighted by the number of MHz-Pops they cover. Thus, the unit price of a combination of a bid of $5/MHz-Pop for one license with the bid of $2/MHz-Pop for a license covering half as many MHz-Pops would be $4/MHz-Pop, 2/3 of $5 plus 1/3 of $2.

[14] A constraint linking multiple bids together could result in multiple bids having fractional values. Another possibility giving rise to multiple bids at the marginal price is that there are separate, unlinked bids at exactly the same price. The reasons for rejecting the marginal bids and not going on to lower bids is the same as the reasons discussed above in the context of the simplified auction. The reason for rejecting all marginal bids is that bidders are free to use many significant digits in their bids. Thus, bids by separate bidders offering the exact same price are suggestive of collusion, and rejecting such bids is a good idea. Bidders can avoid equality in their own bids by adding or subtracting a different tiny amount to each bid, so there should be no problem in rejecting unlinked bids from one bidder with the same unit price.

combination. Suppose for example, a bidder has offered $2/ MHz-pop for licenses A and B and also offered $1.80 per MHz-pop for license C linked to different bids for A and B at $1.80 per MHz-pop; it does not make sense to reject its bid for C due to the conflict with its own prior bids on the same combination.

Computer scientists may wish to note that since each time the procedure returns to Step 1 at least one bid is permanently discarded, the number of linear programming problems that must be solved can bounded above by the number of bids. Since the worst case bound for computational effort for solving linear programming problems is polynomial, so is the worst case bound on the total amount of computation involved here if the number of bids is limited. Worst case bounds are usually conservative. In this case, they are likely to be extremely conservative. The reason is that what is being sold, in most cases, is relaxation rights on existing licenses. The value of these rights should be higher to the holder of the existing license or to someone with whom she strikes an exclusive deal. Hence, competitive conflicting combinations from *different* bidders should be rare.

An Example

We now present a highly simplified illustrative example. It involves relaxation rights on the 12 licenses shown in Table 2.

Table 2. Licenses for the Example

License #	MHx-Pops ($x10^6$)
1	60.2
2	43.5
3	60.2
4	43.5
5	44.0
6	32.6
7	25.8
8	37.4
9	10.9
10	48.7
11	30.8
12	51.9

These licenses have a total of 488.5 million MHz-pops. We will assume that the sale will try to sell 120 million of these, approximately one quarter of them, and have a 10 percent tolerance, thus allowing sale of up to 132 million MHz-pops.

We assume that there are six potential bidders.

Bidder 1 controls licenses 3 and 4 and would like to change the service offered on them. Doing so on 3 alone is feasible, but doing so on 4 alone is not. His values for relaxation rights are $100 million for 3 and $125 million for licenses 3 and 4. He will make two bids $100 of million for 3. The second of these will be linked to a bid of $25 million for 4. (If he allocated more of the linked pair of bids to the bid on 4, he might increase his chance of winning both, but only when the incremental cost for 4 is more than $25 million.)

Bidder 2 has conditional deals with the holders of licenses 6, 7, and 8. She needs to get rights to just one of these to provide a new service. Her value is $25 million for 6, $20 million for 7, and $30 million for 8. She will make bids in these amounts for the licenses but link each of these three bids to a bid on a "pseudo-item" so as to be sure not to win more than one.

Bidder 3 controls licenses 1 and 2. She currently has no plans to change the service she is offering on them, but would like to lock in future flexibility. She is willing, independently, to pay $10 million for relaxation rights on 1 and $8 million for relaxation rights on 2 and will make separate bids of these amounts.

Bidder 4 is a speculator. She controls no licenses. She decides to bid $1 million each on all licenses except numbers 4 and 9, but she wants to be sure not to spend more than $3 million dollars. Therefore, she will link her ten bids by a budget-like constraint of $3 million. To avoid having her bids for licenses 1 and 3 tied and thus treated as linked, she will add $0.01 to her bid for license 1.[15] In addition, she can only raise enough up-front money to cover deposits for 180 MHz-pops. Hence, she realizes that if

[15]For simplicity of exposition, except for this bid we are using round numbers for bids. In practice, bidders would have an incentive to avoid round numbers to avoid unintended ties and to make their bids unpredictable to their competitors.

she were high bidder on licenses 1, 3 and 12, her bid on license 3, the one with the lowest bid per MHz-pop, would be rejected.

Bidder 5 controls license 5 and is willing to pay $8 million dollars for relaxation rights and will bid this amount.

Bidder 6 controls licenses 10, 11, and 12. He wants to offer a service that will require two of the three licenses and would benefit from the third. His values are $100 million for 10 and 11, $130 million for 10 and 12, $110 million for 11 and 12, and $150 million for all three. Note that the total MHz-pops on the three licenses is just below 132 million, so it is possible for him to win all three. He submits four conflicting bids reflecting these values. He chooses the bid amounts to make the $/MHz-pops the same on all linked bids.

No one bids on license 9.

Table 3 shows the bids.

Table 3 Bids on Licenses in Table 2

Bid #	Bidder	License	Amount (10^6\$)	$/MHz-pop	Linked Bids	Budget Constr.
1	1	3	100	1.661	-	-
2	1	3	100	1.205	3	-
3	1	4	25	1.205	2	-
4	2	6	25	0.767	7	-
5	2	7	20	0.775	7	-
6	2	8	30	0.802	7	-
7	2	P2	0	-	4, 5, 6	-
8	3	1	10	0.166	-	-
9	3	2	8	0.184	-	-
10	4	1	1+	0.017	-	B4
11	4	2	1	0.023	-	B4
12	4	3	1	0.017	-	B4
13	4	5	1	0.023	-	B4
14	4	6	1	0.031	-	B4
15	4	7	1	0.039	-	B4
16	4	8	1	0.027	-	B4
17	4	10	1	0.021	-	B4
18	4	11	1	0.032	-	B4
19	4	12	1	0.019	-	B4
20	5	5	8	0.182	-	-
21	6	10	61.3	1.258	22	-
22	6	11	38.7	1.258	21	-
23	6	10	62.9	1.292	24	-
24	6	12	67.1	1.292	23	-
25	6	11	41	1.330	26	-
26	6	12	69	1.330	25	-
27	6	10	55.6	1.142	28, 29	-
28	6	11	35.2	1.142	27, 29	-
29	6	12	59.2	1.142	27, 28	-

The winner determination calculation proceeds directly to Step 4. In it, bid 1 by bidder 1 is honored. Bids 25 and 26 of bidder 6 are the marginal bids, and they cannot be honored within the tolerance, but they set the price at $1.33/MHz-pop. Thus, bidder 1 pays $80.066 million for the 60.2 MHz-pops of relaxation rights on license 1. Nothing more is sold. (In a larger, more realistic example, a larger fraction of the target quantity is likely to be sold.) All unsold relaxation rights will be offered again in next year's auction.

4. Discussion

Radio spectrum is a highly valuable national asset. There are strong arguments that US spectrum is badly under used and over restricted and that a system based upon expanded property rights would work better. While there is a legitimate need to protect temporarily non-licensees who have invested in equipment, the overriding picture is one of misallocation and of the use of administrative procedures to block competition. The proposal in this paper would gradually make spectrum available on a property-rights-like basis. Its gradual nature is an advantage. It will take a while for capital markets and physical ones to adapt, and non-licensee purchasers of equipment will have a chance for their equipment to be used. The use of competition to determine which spectrum is freed up first will tend to assure that the spectrum first released from usage restrictions goes to meet the most pressing unmet needs.

The auction is not "optimal," but it is reasonable. It should prove to be workable for fairly large auctions. It should allow bidders to represent important synergies. It should give good incentive signals to bidders whenever the chance that a given bid will be the marginal one is small. It should be relatively resistant to collusion. It should work particularly well in a situation in which one party already controls some rights on each license and thus is the only party bidding for relaxation rights on it.

The process should efficiently pick out to sell first the most valuable rights. No administrative determination will be needed. Nonetheless, critical spectrum that should not be offered can be protected.

The auction allows combinatorial bidding. Nonetheless, there will not be computational problems since binding integer constraints will result in bids being discarded. The computational effort of the winner determination problem is bounded by solving at most one LP for each bid, and that bound is unlikely to be at all close.

The proposed process is independent of the application of the funds it generates. If desired, some of the funds can be used to compensate for dropped public interest obligations. In the past, the FCC has received less for some licenses than it might have because independent companies formed coalitions before entering the auction. This happened to an extreme extent in some European spectrum sales. Hence, it is tempting to suggest that legislation enabling the auctions should protect competitiveness by restricting joint bidding, not just by coalitions formed after bid deposits have been made, but also by joint ventures formed after the legislation is introduced. However, some new uses of spectrum may well require rights held by different parties. In such cases, coalition formation is natural and can be helpful. The solution is for the FCC to limit the amount of spectrum to be sold so that there is a high "eligibility ratio,"—i.e., there are four or more serious bidders for each license that is to be sold. This should ensure that there is serious competition even in the face of coalitions and discourage coalitions that would only pay off by reducing competition in order to drop prices.

To reiterate the main point of this paper, under the proposal it makes, no licensee's rights will be damaged or limited in any way, but no licensee or other party will get spectrum rights without competition. In particular, those with rights for a use that greatly under uses spectrum will have to compete with others to get their license restrictions eased even though they may be the only bidder for the particular rights that compliment theirs. **It is not necessary to give away spectrum rights in order to have the advantages of private ownership incentives**

Appendix

This Appendix present a mathematical formulation of the optimization problem discussed above in Section 3. It assumes that all bids below the minimum allowable bid have already been deleted. It also assumes that only authorized bids on pseudo-items are included and that the size of each pseudo-item is defined as 0.

Let i index licenses, j index bids, k index bidders, and c index combinatorial bids. If bidder k makes bid j on license i, let b_{ijk} be the price per MHz-Pop offered by that bid. Let x_{ij} be 1 if bid j is for license i, and 0 otherwise; and let X_{jk} be 1 if bid j is by bidder k, and 0 otherwise. Let s_i be the size of license i measured in MHz-Pops. Let y_j be the fraction of bid j that wins. Let z_c be the fraction of combinatorial bid c that wins, and let $J(c)$ be the set of bids included in combinatorial bid c. Let S be the number of MHz-Pops that are scheduled to be sold. Let E_k be the eligibility of bidder k. Let B_k be the budget limit of bidder k (infinite if bidder k specifies no budget limit).

The optimization problem is

Maximize $Z = \Sigma_{ijk} b_{ijk} s_i y_j$

Subject to $\Sigma_{ij} s_i y_j \leq S$,

$\Sigma_j x_{ij} y_j \leq 1$, for all i,

$\Sigma_{ij} s_i x_{ij} X_{jk} y_j \leq E_k$, for all k,

$\Sigma_{ij} s_i x_{ij} X_{jk} b_{ijk} y_j \leq B_k$, for all k,

$z_c = y_j$, for all j in set $J(c)$, and for all c,

$0 \leq y_j \leq 1$, for all j,

y_j integer, for all j.

The first constraint assures that the relaxation rights to no more than the allowed number of MHz-Pops is sold. The second set of constraints assures that the relaxation rights to no license is sold more than once. The third set of constraint assures that no bidder exceed her eligibility. The fourth set assures that no bidder exceeds her budget. The fifth set ties together the parts of combinatorial bids. The sixth set of constraints assures that the fraction of each bid that is accepted lies in the range [0,1]. (Because of the second set of constraints, the upper bound in this set of constraints is redundant.) The final constraint changes what would otherwise be a linear programming problem into an integer programming problem by forcing the fraction of each bid accepted to be either 0 or 1.

Acknowledgement

We are grateful for extremely helpful comments from Professor Sunju Park and from Michael Calabreseand members of his program at the New America Foundation including Troy Kravitz and Jim Snider.

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